Control Design for A Class of Nonlinear Systems Based on External Disturbance Observer

Dao Phuong Nam
Ha Noi University of
Science and Technology
01, Dai Co Viet, Hanoi, Vietnam
nam.daophuong@hust.edu.vn

Do Trong Tan
Ha Noi University of
Science and Technology
01, Dai Co Viet, Hanoi, Vietnam

Bui Minh Thang
Ha Noi University of
Science and Technology
01, Dai Co Viet, Hanoi, Vietnam
20153487@student.hust.edu.vn

Nguyen Truong Thanh
Ha Noi University of
Science and Technology
01, Dai Co Viet, Hanoi, Vietnam
20153401@student.hust.edu.vn

ABSTRACT

This paper presents some analysis about the essential role of finite time convergence in sliding mode control and effects of selected parameters on control performance. The main technique of this paper based on properties of differential equation. Moreover, several exploration in depending of parameters are given out. Simulation results pointed out the good behaviour of the proposed methods for flexible joint manipulator systems.

CCS Concept:

Computing methodologies → Systems theory

Keywords

Disturbance Observer (DO), Sliding Mode Control (SMC), Stability.

1. INTRODUCTION

The Almost industrial systems are affected by external disturbances, such as manipulator control systems, robotic systems, etc. Many control schemes have been utilized based on robust adaptive control without estimating disturbance [5-9]. Beside removing disturbances that probably cannot measuring, we estimate them to design sliding mode control based disturbance observer (DO) [2]. However, authors in [2] have not discussed about the efficiency of parameters in control performance. In [3], the proposed controller ensures the improvement of disturbance attenuation. In contrast, the reduction of computation amount is the important task. Besides, external disturbance mentioned depend only on time. Our work uses the sliding mode control to simplify the computation amount and give out some analysis of parameters' effect on control performance. The paper is organized as follows: The second section, we focus on problem statements. Next section, we pay attention to design the proposed controller. The fourth section, we present the analysis about efficiency of parameters. The fifth section, simulation results provide evidences for these analysises. Final section is the conclusion of the brief.

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2. PROBLEM STATEMENTS

Consider a class of nonlinear system is described as follows [2]:

$$\begin{cases} \dot{x}_1 = \dot{x}_2 + d_1(x,t) \\ \dot{x}_2 = \dot{x}_3 + d_2(x,t) \\ \quad . \\ \quad . \\ \quad . \\ \dot{x}_{n-1} = x_n + d_{n-1}(x,t) \\ \dot{x}_n = a(x) + b(x)u + d_n(x,u,t) \\ y = x_1 \end{cases} \tag{1}$$

where $x = \left[x_1\ x_2\ \ldots\ x_n \right]^T \in \mathbb{R}^n$ is the vector of state variables, $u \in \mathbb{R}$ is the control input and $y \in \mathbb{R}$ is the output signal. The external disturbance $d_i(x,t)\ \ i = 1,2,\ldots,n-1$ does not depend on control input, while the disturbance $d_n(x,u,t)$ depends on $u \in \mathbb{R}$. $a(x), b(x)$ are continuous functions.

Assumption 1: Every disturbances are continuous and satisfy the inequalities:

$$\left| \frac{d^j d_i(x,t)}{dt^j} \right| \le \mu\ i = 1,2,\ldots,n;\ j = 0,1,\ldots,r$$

The control objective is to find the control input to obtain that the output signal converges to attraction region in presence of external disturbances.

3. EXTERNAL DO BASED CONTROL

Initially, we concentrate on designing the DO as described:

The system (1) is affected by disturbances $d_i(x,t)$ $i = 1,2,...,\mathrm{n}-1$, we implement the observer to estimate disturbances based on the following formulas as described in [2]:

$$\begin{aligned} &\hat{d}_i^{(j-i)} = p_{ij} + l_{ij}x_i \qquad i = 1,2,...,n-1 \\ &\dot{p}_{ij} = -l_{ij}(x_{i+1} + \dot{\hat{d}}_i) + \hat{d}_i^{(j)} \quad j = 1,2,...,r-1 \qquad (2) \\ &\dot{p}_{ir} = -l_{ir}(x_{i+1} + \hat{d}_i) \end{aligned}$$

On the other hand, towards the component $d_n(x,u,t)$, the corresponding observer has been pointed out [2]:

$$\begin{aligned} &\hat{d}_n^{(j-1)} = p_{nj} + l_{nj}x_n \\ &\hat{p}_{nj} = -l_{nj}(a(x) + b(x)u + \hat{d}_n) + \hat{d}_n^{(j)} \quad j = 1,2,...,(r-1)(3) \\ &\dot{p}_{nr} = -l_{nr}(a(x) + b(x)u + \hat{d}_n) \end{aligned}$$

where p_{ij} are auxiliary variables and l_{ij} are arbitrary positive constants ensuring that

$$D_i = \begin{bmatrix} -l_{i1} & 1 & 0 & ... & 0 \\ -l_{i2} & 0 & 1 & ... & 0 \\ . & . & . & . & . \\ . & . & . & . & . \\ . & . & . & . & . \\ -l_{i(r-1)} & 0 & 0 & ... & 1 \\ -l_{ir} & 0 & 0 & ... & 0 \end{bmatrix}$$ have all eigenvalues belonging to the left side of complex coordinate plane.

Remark 1: We absolutely adjust the speed of observer error by selecting the eigenvalues.

Remark 2: it is necessary to ensure that the time of convergence of sliding surface is finite. The fact is described based on the following example:

We consider the system as follows:

$$\begin{cases} \dfrac{dx}{dt} = Ax + Bs \\ \dfrac{ds}{dt} = Cx + Ds \end{cases} \qquad 4$$

where:

$A \in \mathbb{R}^{n\times n}, B \in \mathbb{R}^{n\times r}, C \in \mathbb{R}^{r\times n}, D \in \mathbb{R}^{r\times r}$, s is the sliding surface. Selecting A is Hurwitz matrix and $\begin{bmatrix} A & B \\ C & D \end{bmatrix}$ is not Hurwitz matrix. We obtain that although s converges to 0 in infinite time, x does not converge to 0.

In [2], the sliding surface has been selected $\sigma = \sum_{i=1}^{n} c_i x_i + \sum_{i=1}^{n-1}\sum_{j=1}^{n-1} c_{j+1}\hat{d}_i^{(j-i)}; c_n = 1$

and the modified sliding surface: $\sigma^* = \sigma - \sigma(0)e^{-\alpha t}$

where α is arbitrary positive constant.

From the estimation (2) and (3), the control input has been proposed in [2]:

$$u = -\frac{1}{b(x)}\left[a(x) + \hat{d}_n + \alpha\sigma(0)e^{-\alpha t} + \sum_{i=1}^{n} c_i(x_{i+1} + \hat{d}_i) + k_l\sigma^* + k_s sat\sigma^* \right](5)$$

$k_l, k_s > 0$

where $sat\sigma^* = \begin{cases} \operatorname{sgn}\sigma^* & khi \ \left|\sigma^*\right| > \varepsilon \\ \dfrac{\sigma^*}{\varepsilon} & khi \ \left|\sigma^*\right| \le \varepsilon \end{cases}$

The stability analysis has been proved in [2].

4. EFFICIENT OF PARAMETERS ON CONTROL SYSTEM PERFORMANCE

We propose the analysis of parameters' efficiency on control system as follows:

- The term $\dfrac{-1}{b(x)}k_l\sigma^*$ guarantee the chatter reduction combine with selecting suitable value k_s described as follows:

  $$\left|\sigma^*\right| \le \lambda_2 = \frac{A\mu + nB\lambda_1 - k_s}{k_l}$$

  Where A is defined as the sum of coefficients $d_i^{(j)}$ obtained by:

  $$\begin{aligned} &\dot{\sigma}^* = -k_l\sigma^* - k_s sat(\sigma^*) + \\ &\sum_{i=1}^{n-1}\sum_{j=i}^{n-1} c_{j+1}(d_i^{(j-i+1)} - \tilde{d}_i^{(j-i+1)}) + \sum_{i=1}^{n-1}\left[c_i + \sum_{j=1}^{n-1} c_{j+1}l_{i(j-i+1)} \right]\tilde{d}_i \end{aligned}$$

  B is the maximum value among the coefficients $\hat{d}_i^{(j)}$
- The selections of positive constants l_{ij} $i = 1,2,...,n$ $j = 1,2,...,n$ in order to obtain the matrix D_i have all eigenvalues which are far from imaginary axis. It can be clearly seen that the higher distance would certainly increase the convergence speed of estimation errors.
- The increasing of coefficient c_i makes to increase the convergence speed. However, the chatter of modified surface goes up respectively.

5. SIMULATION RESULTS

We implement simulations in the two cases:

In the first case, we consider the following system being the individual case of nonlinear system (1):

$$\begin{cases} \hat{x}_1 = x_2 + d_1(x,t) \\ \hat{x}_2 = -6x_1 - 5x_2 + \cos(x_1) + u \\ y = x_1 \end{cases} \qquad (7)$$

where:

$$d_1 = x_2\sin(t) + x_1^{\,2};\;\; x(0) = \begin{bmatrix}0.1 & 0\end{bmatrix}^T;$$
$$k_l = 10;\; k_s = 0;\; c_1 = 10;\; c_2 = 1; \alpha = 1.$$
$$l_{11} = 100; l_{12} = 20;$$

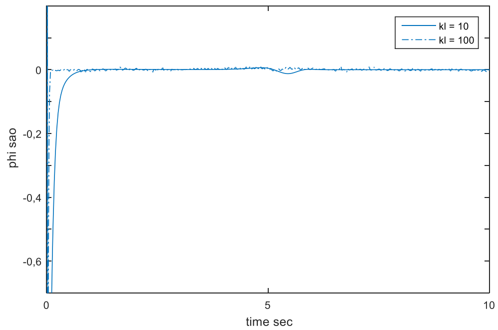


Figure 1. The dynamic of modified sliding surface

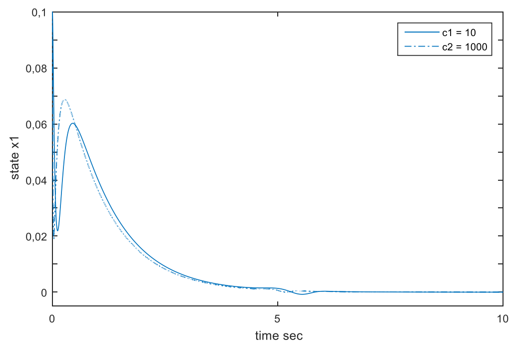


Figure 2. The behaviour of output signal corresponding to variation of c

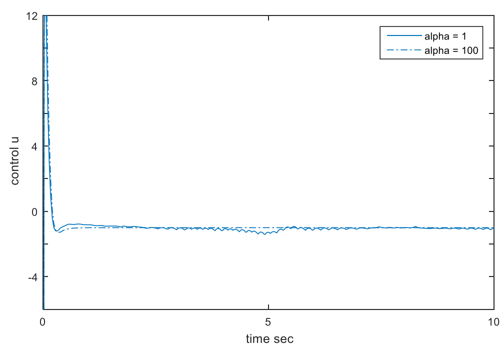


Figure 3. The behaviour of input control signal corresponding to variation of alpha

Simulation results show good behaviour of sliding surface and input, output (Fig. 1,2,3). In Fig. 1, the first result show that when k_l increases from 10 to 100, the border of attraction region of modified sliding surface is smaller but the level of chatter goes up. The second result describes when c_1 increases from 10 to 1000, the output signal converge to attraction region faster. The last result is clearly seen that α has a significant change from 1 to 100, the chatter of the input control will be reduced.

In the second case, we consider the flexible joint manipulator being described by the following equations:

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = x_3 - \dfrac{MgL}{I}\sin(x_1) - \dfrac{K}{I}x_1 \\ \dot{x}_3 = x_4 \\ \dot{x}_4 = \dfrac{K}{J}x_1 - \dfrac{I}{J}x_3 + \dfrac{u}{J} \end{cases} \tag{8}$$

However, because the manipulator is affected by external disturbance and uncertainties, therefore we obtain new dynamic equations:

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = x_3 - \dfrac{MgL}{I}\sin(x_1) - \dfrac{K}{I}x_1 + \Delta\dfrac{MgL}{I}\sin(x_1) + \Delta\dfrac{K}{I}x_1 \\ \dot{x}_3 = x_4 \\ \dot{x}_3 = \dfrac{K}{J}x_1 - \dfrac{I}{J}x_3 + \dfrac{u}{J} + \Delta\dfrac{K}{J}x_1 + \Delta\dfrac{I}{J}x_3 + \Delta\dfrac{u}{J} + \dfrac{v}{J} \end{cases} \tag{9}$$

where $-0.1 \le \Delta \le 0.1$ is described by parameter error, v is the external disturbance of input. Using

$$\begin{cases} d_2(x) = -\dfrac{MgL}{I}\sin(x_1) - \dfrac{K}{I}x_1 + \Delta\dfrac{MgL}{I}\sin(x_1) + \Delta\dfrac{K}{I}x_1 \\ d_4(x,t) = \Delta\dfrac{K}{J}x_1 + \Delta\dfrac{I}{J}x_3 + \Delta\dfrac{u}{J} + \dfrac{v}{J} \\ a(x) = \dfrac{K}{J}x_1 - \dfrac{I}{J}x_3 \\ b(x) = \dfrac{1}{J} \end{cases} \tag{10}$$

Based on the general design in (5) we obtain the control input as follows:

Step 1: Select the DO

We can implement the order 2 or 3 DO and the order 3 is better. So that, we obtain result:

$$d_2: \begin{cases} \hat{d}_2 = p_{21} + l_{21}x_2 \\ \dot{p}_{21} = -l_{21}(x_3 + \hat{d}_2) + \hat{\dot{d}}_2 \\ \hat{\dot{d}}_2 = p_{22} + l_{22}x_2 \\ \dot{p}_{22} = -l_{22}(x_3 + \hat{d}_2) + \hat{\ddot{d}}_2 \\ \hat{\ddot{d}}_2 = p_{23} + l_{23}x_2 \\ \dot{p}_{22} = -l_{23}(x_3 + \hat{d}_2) \end{cases} \tag{11}$$

$$\text{and } d_4: \begin{cases} \hat{d}_4 = p_{41} + l_{41}x_4 \\ \dot{p}_{41} = -l_{41}(a(x) + b(x)u + \hat{d}_4) + \hat{\dot{d}}_4 \\ \hat{\dot{d}}_4 = p_{42} + l_{42}x_4 \\ \dot{p}_{42} = -l_{42}(a(x) + b(x)u + \hat{d}_4) + \hat{\ddot{d}}_4 \\ \hat{\ddot{d}}_4 = p_{43} + l_{43}x_4 \\ \dot{p}_{23} = -l_{23}(a(x) + b(x)u + \hat{d}_4) \end{cases} \tag{12}$$

Step 2: Sliding Surface and control input

The sliding surface and modified sliding surface are selected as follows [2]:

$$\sigma = c_1x_1 + c_2x_2 + c_3x_3 + x_4 + c_3\hat{d}_2 + \dot{\hat{d}}_2 \quad (13)$$

$$\sigma^* = c_1x_1 + c_2x_2 + c_3x_3 + x_4 + c_3\hat{d}_2 + \dot{\hat{d}}_2 - \sigma(0)e^{-\alpha t} \quad (14)$$

The control input is obtained:

$$u = -\frac{1}{b(x)}\Big[a(x) + \hat{d}_4 + \alpha\sigma(0)e^{-\alpha t} + c_1(x_2 + \hat{d}_1) + c_2(x_3 + \hat{d}_2) + c_3(x_4 + \hat{d}_3) + k_l\sigma^* + k_s sat(\sigma^*)\Big] \quad (15)$$

We have $|\sigma^*| \le \lambda_2$ (16)

with: $$\lambda_2 = \frac{\lambda_1(c_2 + c_3l_{21} + c_4l_{22} + 2c_3) + 2c_3\mu - k_s}{k_l} \quad (17)$$

We absolutely obtain arbitrary small attraction region by using the suitable parameters.

Step 3: Algorithm

a. Find the parameters of (1)
b. Estimate the disturbance of input
c. Selecting $l_{21}, l_{22}, l_{23}, l_{41}, l_{42}, l_{43} > 0$ satisfy matrix $\begin{bmatrix} -l_{21} & 1 & 0 \\ -l_{22} & 0 & 1 \\ -l_{23} & 0 & 0 \end{bmatrix}$ and $\begin{bmatrix} -l_{41} & 1 & 0 \\ -l_{42} & 0 & 1 \\ -l_{43} & 0 & 0 \end{bmatrix}$ have eigenvalue belonging the left side of complex coordinate plane. Selecting $\{s_1, s_2, s_3\}$ are positive and big enough. From the equations:
$(s+s_1)(s+s_2)(s+s_3)$
$= s^3 + l_{21}s^2 + l_{22}s + l_{23}$
$= s^3 + l_{41}s^2 + l_{42}s + l_{43}$
We have $\{l_{21}, l_{22}, l_{23}, l_{41}, l_{42}, l_{43}\}$
d. Selecting $\{s_4, s_5, s_6\}$ are positive and big enough. From the equation:
$(s+s_4)(s+s_5)(s+s_6) = s^3 + c_3s^2 + c_2s + c_1$
we obtain $\{c_1, c_2, c_3\}$
e. Selecting $|\Delta| < 0.1, k_s > 0$, $\alpha > 0$ big enough and $k_l > 0$ is bigger than l_{ij}, c_i

Finally, we obtain the control law and simulation results based on parameters (Table 1) as follows:

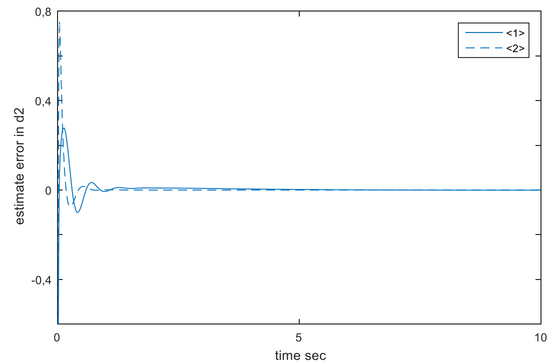
I	J	M	g	L	K
Inertia moment of joint	Inertia moment of motor	Mass of Link	Gravity	Length of Link	Stiffness
$kg.m^2$	$kg.m^2$	kg	$m.s^{-2}$	m	N.m/rad
1	2	1	10	2	100

(Information of parameters and its value)

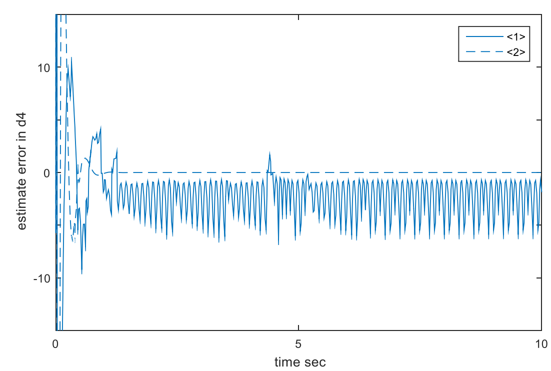
When eigenvalues of D_i matrix change, the estimation error in d_2 and d_4 are reduced. Because in [2], we have equation:

$$\dot{\tilde{e}}_i = D_i\tilde{e}_i + E_id_i^{(r)} \quad (18)$$

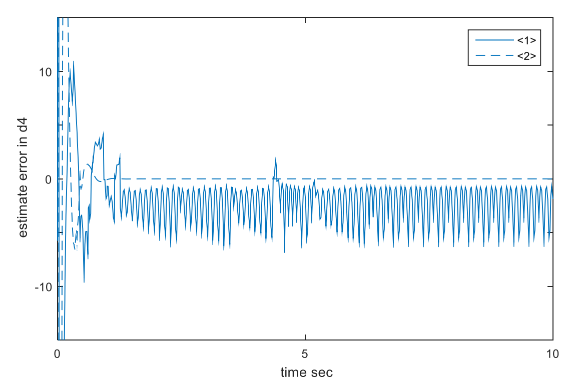
So that, the estimation error in d_2 and d_4 are reduce if the eigenvalues less than 0 and far from imaginary axis.



(Fig 4a)



(Fig 4b)



(Fig 4c)

Figure 4. Result of change l_{ij}

$<1>$ $l_{21} = l_{41} = 100, l_{22} = l_{42} = 20$
$<2>$ $l_{21} = l_{41} = 100, l_{22} = l_{42} = 1875$

6. CONCLUSIONS

This work determines the role of finite time convergence of sliding surface in sliding mode control law. Moreover, the analysis of parameters' efficiency is considered. Simulation results demonstrate the proposed contents.

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